Math 1050–4 Midterm Exam #2

March 13, 2015

- No notes, books, etc. are allowed.
- Answer the questions in the spaces provided.
- You have 50 minutes to complete this exam.
- **Good luck!**

Name: _____ uNID: _____

True/False

1. (8 points) For #1–8, write the word "True" or "False."

(a) $\sqrt[n]{x+y} = \sqrt[n]{x} + \sqrt[n]{y}$ ← Let $x = 1$ & $y = 1$ (as an example) (a) False

(b) $(xy)^n = x^n y^n$ (b) True

(c) $a(b+c) = ab + ac$ (c) True

(d) $(x+y)^n = x^n + y^n$ (d) False

(e) $\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$ (e) True

(f) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ (f) True

(g) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$ (g) True

(h) $-7x^5 + 3x^4 - 2x + 1$ has 7 roots (h) False

Degree = 5 ⇒ maximum number of roots possible is 5.

Algebra

2. (1 point) Find x where $(x+16)^3 + 9 = -18$. (Your answer should be an integer in standard form.)

$(x+16)^3 + 9 = -18$

$(x+16)^3 = -18 - 9$

$(x+16)^3 = -27$

$x + 16 = \sqrt[3]{-27}$

$x + 16 = -3$

$x = -16 - 3$

$x = -19$

2. -19

3. (1 point) If $g(x)$ is an invertible function, and $g(-3) = 5$, then what is $g^{-1}(5)$?

$g(-3) = 5$

$\rightarrow -3 = g^{-1}(5)$

3. -3

Points earned: _____
out of a possible 10 points

4. (1 point) Find the inverse of $f(x) = 9\sqrt[3]{x-1} + 2$. (You can check your answer by seeing if $f^{-1} \circ f(x) = x$.)

$y = 9\sqrt[3]{x-1} + 2$

$y - 2 = 9\sqrt[3]{x-1}$

$\frac{y-2}{9} = \sqrt[3]{x-1}$

$\left(\frac{y-2}{9}\right)^3 = x - 1$

$\left(\frac{y-2}{9}\right)^3 + 1 = x$

$f^{-1}(y) = \left(\frac{y-2}{9}\right)^3 + 1$

4. $f^{-1}(y) = \left(\frac{y-2}{9}\right)^3 + 1$

5. (1 point) What is the implied domain of $g(x) = -\sqrt[3]{4x-9} + 3x^7 - 2$? (Write your answer as an interval.)

$4x - 9 < 0$: values of x for which $g(x)$ does not make sense

$4x < 9$

$x < 9/4 \longleftrightarrow (-\infty, 9/4)$

$\mathbb{R} - (-\infty, 9/4)$: all values of x for which $g(x)$ does makes sense (i.e. its implied domain

equivalently: $[9/4, \infty)$

5. $\mathbb{R} - (-\infty, 9/4)$

or: $[9/4, \infty)$

6. (1 point) Suppose that $a \neq 0$ and that $b^2 - 4ac \geq 0$. Write the following as an integer in standard form:

2. This is $p(x)$ evaluated at its root

$$a\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)^2 + b\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) + c = 0$$

1. This is a root of the polynomial $p(x) = ax^2 + bx + c$

6. 0

Points earned: _____
out of a possible 3 points

7. (1 point) Find $\dfrac{8x^4 + 6x^3 - 6x^2 - 2x + 4}{2x^2 + 1}$

$$
\begin{array}{r|l}
 & 4x^2 \quad 3x \quad -5 \\
\hline
2x^2+1 & 8x^4 + 6x^3 - 6x^2 - 2x + 4 \\
 & -(8x^2 \qquad\quad + 4x^2 \qquad\quad) \\
\hline
 & 6x^3 - 10x^2 - 2x + 4 \\
 & -(6x^3 \qquad\quad + 3x \quad) \\
\hline
 & -10x^2 - 5x + 4 \\
 & -(-10x^2 \qquad\quad -5) \\
\hline
 & -5x + 9
\end{array}
$$

7. $4x^2 + 3x - 5 + \dfrac{-5x+9}{2x^2+1}$

Solution to #7: $4x^2 + 3x - 5 + \dfrac{-5x+9}{2x^2+1}$

8. (1 point) Find $\dfrac{3x^3 + 7x^2 - 2}{x + 3}$

$$= \frac{3x^3 + 7x^2 + 0x - 2}{x - (-3)}$$

$$
\begin{array}{r|rrrr}
-3 & 3 & 7 & 0 & -2 \\
 & & -9 & 6 & -18 \\
\hline
 & 3 & -2 & 6 & -20
\end{array}
$$

8. $3x^2 - 2x + 6 + \dfrac{-20}{x+3}$

9. (1 point) What is the slope of the straight line in $\mathbb{R}^2$ that passes through the point $(1, 3)$ and $(6, 13)$?

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 3}{6 - 1} = \frac{10}{5} = 2$$

or

$$= \frac{3 - 13}{1 - 6} = \frac{-10}{-5} = 2$$

9. 2

Points earned: _____
out of a possible 3 points

10. (1 point) Complete the square: Write $6x^2 - 6x + 5$ in the form $\alpha(x+\beta)^2 + \gamma$, where $\alpha, \beta, \gamma \in \mathbb{R}$.

$\alpha = a$

$\beta = b/2a$

$\gamma = c - b^2/4a$

or: $6(x^2 - x + (\frac{-1}{2})^2) + 5 - 6(\frac{-1}{2})^2$

$= 6(x^2 - x + \frac{1}{4})^2 + 5 - \frac{6}{4}$

$= 6(x - \frac{1}{2})^2 + (5 - \frac{3}{2})$

$\rightarrow \begin{cases} \alpha = 6 \\ \beta = -6/2(6) = -1/2 \\ \gamma = 5 - (-6)^2/4(6) = 5 - \frac{36}{4(6)} = 5 - 3/2 \end{cases}$

10. $6(x - 1/2)^2 + (5 - \frac{3}{2})$

11. (1 point) How many roots does $-5x^2 + 6x - 3$ have?

$b^2 - 4ac$

$= (6)^2 - 4(-5)(-3)$

$= 36 - 4(5)(3)$

$= 36 - 20(3)$

$= 36 - 60 < 0 \Rightarrow$ no roots

11. no roots

12. (1 point) Find the roots of $x^2 - 4x - 10$

$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4) + \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}$

$= \frac{4 + \sqrt{16 + 40}}{2} = \frac{4 + \sqrt{56}}{2}$

$\begin{array}{r} 19 \\ 2\overline{)28} \\ 2\overline{)56} \end{array}$

$56 = 2^2(19)$

$\sqrt{56} = 2\sqrt{19}$

$= \frac{4 + 2\sqrt{19}}{2} = 2 + \sqrt{19}$

12. $2 + \sqrt{19}$ & $2 - \sqrt{19}$

13. (1 point) Find a root of $6x^3 + 12x^2 + 9x + 3$

Factors of 3: 1, 3, -1, -3

$x = 1$: $6(1)^3 + 12(1)^2 + 9(1) + 3 \neq 0$

$x = 3$: $6(3)^3 + 12(3)^2 + 9(3) + 3 \neq 0$

$x = -1$: $6(-1)^3 + 12(-1)^2 + 9(-1) + 3$

$= -6 + 12 - 9 + 3$

$= 6 - 6$

$= 0$

13. -1

Points earned: _____
out of a possible 4 points

14. (1 point) Completely factor $3x^3 - 4x^2 + x - 10$. (Hint: 2 is a root.) (Your answer should be a product of a constant polynomial (a number), and perhaps some monic linear polynomials (monic i.e. the leading coefficient equals to 1) and monic quadratic polynomials (which have no roots). (Let $p(x) = 3x^3 - 4x^2 + x - 10$)

1.

$$\begin{array}{r|rrr|r} 2 & 3 & -4 & 1 & -10 \\ & & 6 & 4 & 10 \\ \hline & 3 & 2 & 5 & 0 \end{array}$$

$\longrightarrow p(x) = (x-2)(3x^2+2x+5)$

2. $b^2 - 4ac = (2)^2 - 4(3)(5)$
$= 4 - 60$
$= -56 < 0$

$\longrightarrow 3x^2+2x+5$ has no roots

3. $p(x) = (x-2)(3)\left(x^2 + \frac{2}{3}x + \frac{5}{3}\right)$

3a. $p(x) = 3(x-2)\left(x^2 + \frac{2}{3}x + \frac{5}{3}\right)$

14. $3(x-2)\left(x^2 + \frac{2}{3}x + \frac{5}{3}\right)$

15. (1 point) Completely factor $2x^3 + 9x^2 + 7x - 6$. (Hint: -2 is a root.) (Your answer should have the same form as described in #21.) (Let $p(x) = 2x^3 + 9x^2 + 7x - 6$)

1.

$$\begin{array}{r|rrr|r} -2 & 2 & 9 & 7 & -6 \\ & & -4 & -10 & 6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$\longrightarrow p(x) = (x-(-2))(2x^2+5x-3)$

2. $b^2 - 4ac = (5)^2 - 4(2)(-3)$
$= 25 + 24$
$= 49 > 0$

$\longrightarrow 2x^2+5x-3$ has 2 roots

2a. The roots are

$\frac{-5+\sqrt{49}}{2(2)}$ & $\frac{-5-\sqrt{49}}{2(2)}$

$= \frac{-5+7}{4}$ & $\frac{-5-7}{4}$

$= 2/4 = 1/2$ $\quad = -12/4 = -3$

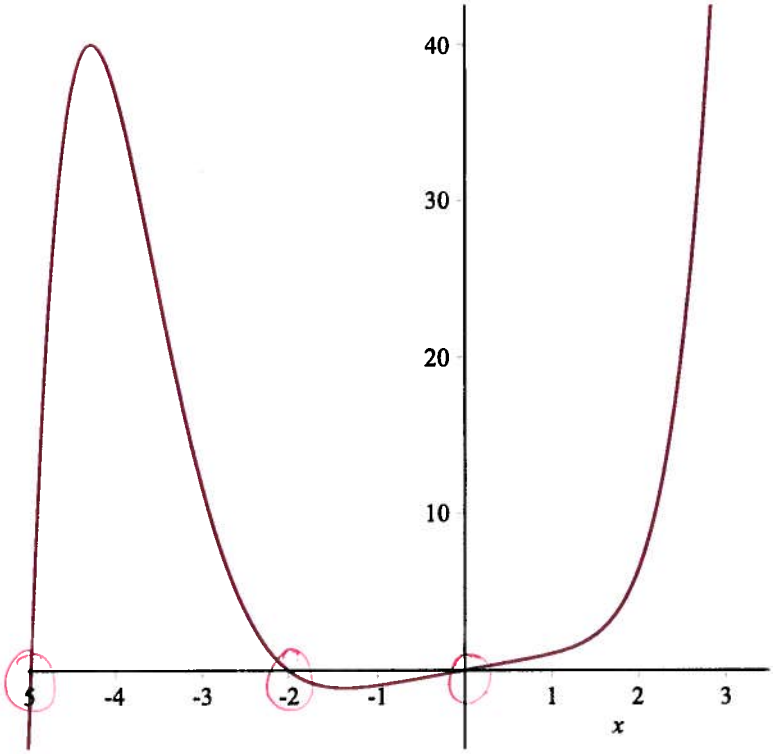
3. $p(x) = (x-(-2))(2)\left(x-\frac{1}{2}\right)(x-(-3))$
$= 2(x+2)(x+3)\left(x-\frac{1}{2}\right)$

15. $2(x+2)(x+3)\left(x-\frac{1}{2}\right)$

Points earned: _____
out of a possible 2 points

Graphs

16. (1 point) List all of the monic linear factors of $p(x)$ that you know of from the graph below.



Roots are -5, -2, 0
(that we know about)

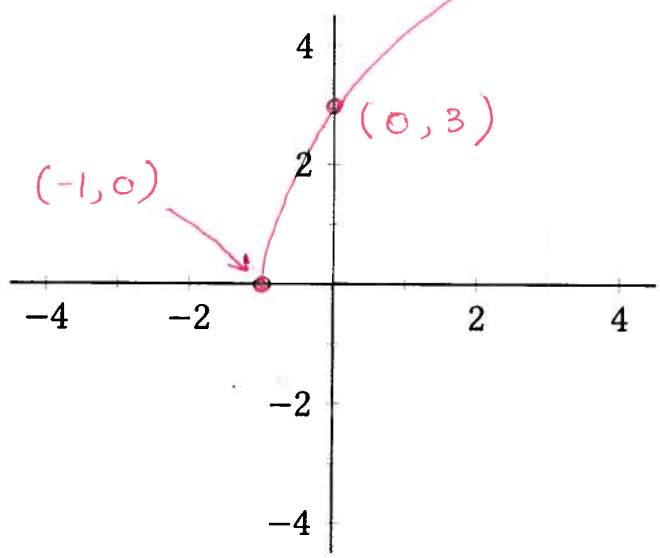
→ Monic linear factors that we know about are:
$x-(-5)$, $x-(-2)$, $x-0$

16. $x+5$, $x+2$, x

Points earned: _____
out of a possible 1 points

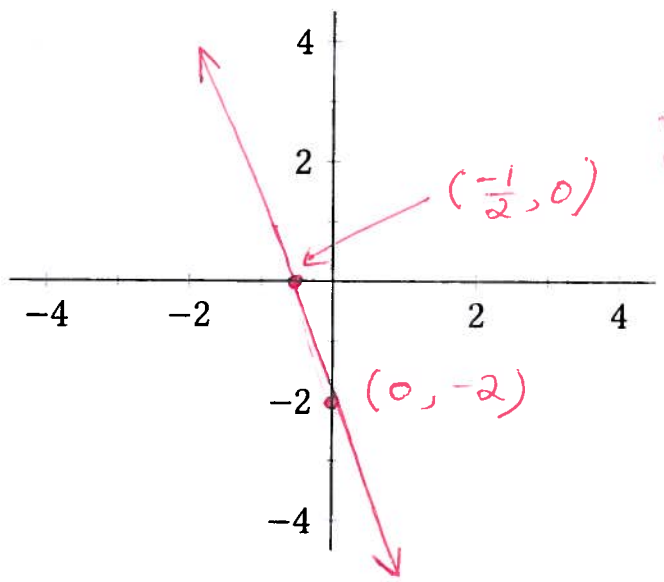
x-intercept: $y=0 \rightarrow$
$\rightarrow x = ?$
$3\sqrt{x+1} = 0 \rightarrow x = -1$

17. (1 point) Graph $3\sqrt{x+1}$, and label its x- and y-intercepts.



y-intercept: $x=0$
$\rightarrow y = 3\sqrt{(0)+1}$
$y = 3$

18. (1 point) Graph $-4x-2$, and label its x- and y-intercepts.



$x=0 \rightarrow y=-2$

$y=0 \rightarrow -4x-2=0$
$-4x = 2$
$x = \frac{-2}{4}$
$x = \frac{-1}{2}$

Points earned: _____
out of a possible 2 points

$y = 0 \rightarrow \sqrt[3]{\frac{x}{3}} - 1 = 0$

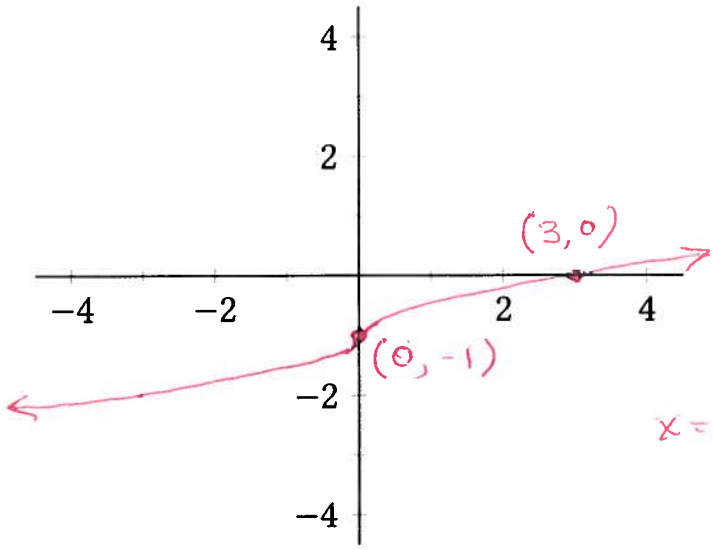
$\sqrt[3]{\frac{x}{3}} = 1$

$\frac{x}{3} = (1)^3$

$x = 3(1)^3$

$x = 3$

19. (1 point) Graph $\sqrt[3]{\frac{x}{3}} - 1$, and label its x- and y-intercepts.



$x = 0 \rightarrow y = -1$

20. (1 point) Graph $-3(x-1)^2 - 1$; label its vertex, and y-intercept. (It does not have an x-intercept.)

Vertex = $(1, -1)$

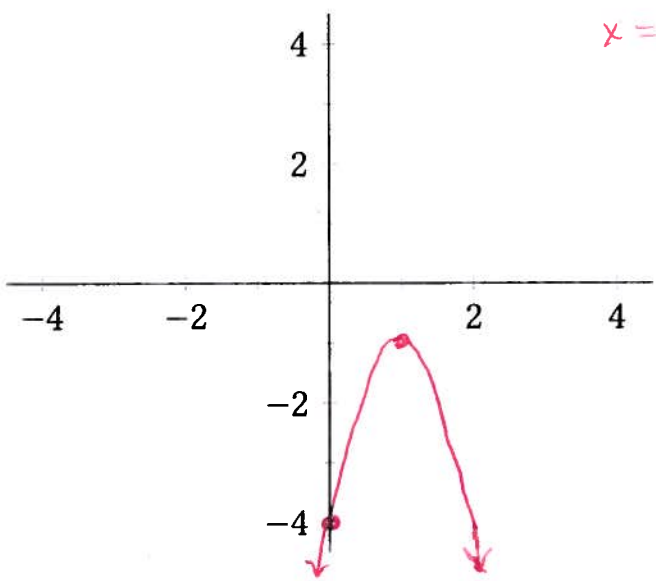
$x = 0 \rightarrow y = -3(0-1)^2 - 1$

$= -3(-1)^2 - 1$

$= -3(1) - 1$

$= -3 - 1$

$= -4$



Points earned: _____
out of a possible 2 points